Problem Set 1

Due October 14

1) Short Answer

a) Derive the shear viscosity and thermal diffusivity of a hard sphere gas by heuristic methods.

b) Consider a heavy particle of mass M radius d_2 in a gas of light particles of mass m, radius d_1 ($d_1 < d_2$, $m \ll M$).

   i) What is the mobility of the heavy particle?

   ii) What is the deflection length for the heavy particle?

   iii) When will heavy particle energy equilibrate with that of a light particle?

c) $K = l_{mfp}/L$, where L is a macroscale (system size) defines the Knudsen number. For $K > 1$, estimate the effective thermal conductivity and shear frictional force/area on its container wall.

2) Assume that the collision term in the Boltzmann equation of an electron system is of the form

$$\left(\frac{\partial f}{\partial t}\right)_{coll.} = -\sum_{\alpha} s_\alpha \int \sigma_\alpha(p,p')dp' \cdot v(p)f(p) + \sum s_\alpha \int \sigma_\alpha(p',p)v(p')f(p')dp'$$

and that the operator D is defined by

$$\left(\frac{\partial f}{\partial t}\right)_{coll.} \equiv -Df = -f(v)\int W(v,v')dv' + \int W(v',v)f(v')dv'$$

where $W(v,v')dv'$ denotes the transition rate of electrons with velocity v into a state with velocity in the interval between $v' + dv'$ (for simplicity, the collisions are assumed to be elastic). Solve the time-dependent Boltzmann equation up through a term linear in E, with the initial condition that E vanishes at $t = -\infty$ and that the electron distribution is the equilibrium distribution f_0, and show that the electric current density at a time t is generally given in the form

$$j_i(t) = \sum_l \int_{-\infty}^{l} E_l(t')dt'\Phi_u(t-t') \qquad (i,l = x,y,z)$$

with

$$\Phi_u(t) = \frac{\langle j_i(t) j_l(0) \rangle}{kT}.$$

Here $\langle\, j_i(t) j_l(0)\, \rangle$ is the correlation function of the electric current, which occurs as a fluctuation in an electron system at equilibrium. Based on this result, express the static and dynamical electric conductivities in terms of the correlation function.

3) Calculate the heat conduction coefficient of a dilute, hard sphere gas using the krook-model collision operator discussed in class.

4) (a) The motion of an electron belonging to a molecule in a rarefied gas may, in some cases, be replaced by that of a harmonic oscillator: it is determined by

$$\frac{dx}{dt} = \frac{p}{m}, \qquad \frac{dp}{dt} = -m\omega_0^2 x - eE(t),$$

where x and p denote the radius vector and the momentum of the electron within the molecule respectively, m the mass, $-e$ the electric charge, ω_0 the characteristic angular frequency, and $E(t)$ an external electric field. Show that the average $\bar{f}(x, p, t)$ of the electron distribution function $f(x, p, t)$, taken over all possible values of the time and of the position at which collisions occur, obeys the equation

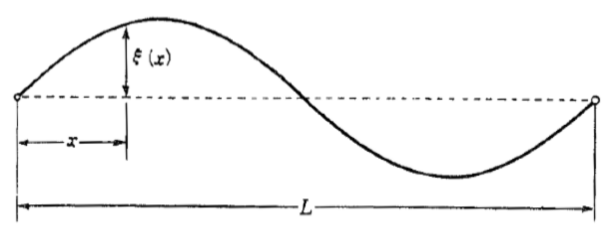
$$\frac{\partial \bar{f}}{\partial t} + \frac{p}{m} \cdot \frac{\partial \bar{f}}{\partial x} + \{-m\omega_0^2 x - eE(t)\} \cdot \frac{\partial \bar{f}}{\partial p} = -\frac{\bar{f} - f_0}{\tau}$$

by assuming that collisions between molecules occur with a mean free flight time τ, and that the electron distribution function immediately after a collision reduces to a given distribution function $f_0(x, p, t)$.

(b) Derive the electric current in the limit of small τ.

(c) What is the effective Ohm's Law for this system? Discuss it.

5) Find the average fluctuation of the deviation of each point of a string stretched between two fixed pins with constant stress, by assuming that the deviation is small compared with the length of string. (Hint: Expand the deviation $\xi(x)$



into a Fourier series, and regard the expansion coefficients $A_1,\ A_2, \ldots$ as the coordinates describing the deviation.)

6) Now, use the Boltzmann Equation to calculate systematically:

(a) the diffusion coefficient for a heavy particle in a gas of light particles

(b) the diffusion coefficient for a light particle in a gas of heavy particles.